

# The Mathematics of Georgia KENO

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## Abstract

The basic mathematics behind the Georgia KENO online game are described using discrete random variables derived from Hypergeometric probability distributions and payoff tables. These new random variables are used to calculate win probabilities for various combinations of the number of spots  $r$  and the number of consecutive draws  $D$ . This information is then used to study the probability landscape resulting in a recommended “best”  $r$  for a particular  $D$ .

## 1 Introduction

KENO<sup>1</sup> is a game of chance in which one tries to match all or part of a group of numbers randomly drawn from an even larger group of numbers. The more numbers that are matched, the higher the payout.

In the simplest case<sup>2</sup>, the *player* or *gambler* fills out a KENO form (or card) (Figure A1) with the following:

1. Number of **spots** to play:  $r \in \{1, 2, 3, \dots, 10\}$ .
2. Numbers to play: user choice of  $r$  distinct numbers in  $1 - 80$ , or sampling  $r$  random numbers from  $1 - 80$ , without replacement (quik pick).
3. Number of consecutive **draws**:  $D \in \{1, 2, 3, 4, 5, 10, 20\}$ . The cost of the card is equal to the number of draws  $D$ .

Once KENO receives its money and the card is run through the system, the player receives a receipt of the wager, which is used to claim winnings (should there be any). Each draw will result in some payoff (which can be zero) to the player. Procedures to calculate the returns based on the options chosen are discussed below.

Note that this document is not meant to encourage or discourage gamblers from playing this game. It is simply meant to inform players of the mathematical laws which govern the game and let them decide on whether to play.

## 2 Single Base Draw

We attempt to put a single base draw on a firm mathematical foundation. A single base draw has  $r$  spots,  $D = 1$  draws, and costs \$1.

Consider a universal set  $U$  consisting of the integers  $1 - 80$ . The size of this set is simply the cardinality of the set (or number of elements in the set):  $|U| = N = 80$ .

A draw consists of the following sequence of events:

1. A player picks (specified or random)  $r$  different numbers from  $U$  resulting in subset  $B$ :  $U \supset B, |B| = r = \# \text{ spots}$ .
2. Twenty numbers are randomly chosen without replacement by KENO from set  $U$  forming subset  $A$ :  $U \supset A, |A| = n = 20$ .

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<sup>1</sup>When we refer to *KENO* we may mean the Georgia Lottery (who run the online game), the proprietor who runs the card and collects the money, or the game itself. Hopefully the context will be clear.

<sup>2</sup>We assume \$1 per draw, no specialty wagering options such as random multipliers on winnings or the BULLS-EYE are selected. Perhaps these will be addressed in a future paper.

The # of matches is simply the cardinality of the set  $A \cap B$  and is a random variable  $X$  distributed hypergeometrically, i.e.,

$$X \sim \text{HYP}(N, r, n).$$

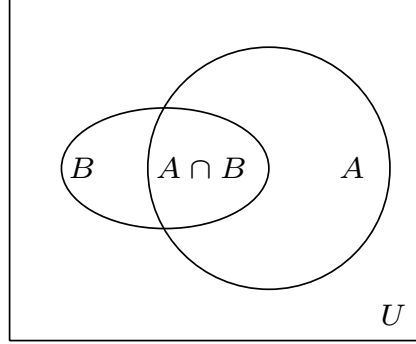


Figure 1: KENO Sets:  $|U| = 80$ ,  $|A| = 20$ ,  $|B| = r = \# \text{ spots}$ ,  $|A \cap B| = x = \# \text{ matches}$

Note that  $|A \cap B|$  can take on any of the values  $0, 1, 2, \dots, r$ . This set theory perspective is illustrated in Figure 1. Substituting values  $N = 80$  and  $n = 20$  gives the probability distribution parametrized by  $r$ :

$$\begin{aligned} \Pr(X = x; N, r, n) &= \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{r}{x} \binom{80-r}{20-x}}{\binom{80}{20}} = \frac{\binom{20}{x} \binom{60}{r-x}}{\binom{80}{r}} = \Pr(X = x; r) \\ \mathbb{E}[X] = \mu &= \frac{nr}{N} = \frac{20r}{80} = \frac{r}{4} \\ \text{Var}[X] = \sigma^2 &= n \left( \frac{r}{N} \right) \left( \frac{N-r}{N} \right) \left( \frac{N-n}{N-1} \right) = 20 \left( \frac{r}{80} \right) \left( \frac{80-r}{8} \right) \left( \frac{80-20}{80-1} \right) = \frac{5r(80-r)}{632} \end{aligned}$$

The probability distributions  $\Pr(x; r)$  are displayed in Table B2 and can be visualized in Figure B1. Computations of  $\mathbb{E}[X]$ ,  $\text{Var}[X]$ , and Shannon Entropy  $H(X)$  for various values of  $r$  are summarized in Table B1.

For each value of the number of spots played  $r$ , there is a payoff function  $f_r(x)$ , where  $x$  is the number of matches and  $x \in \{0, 1, 2, \dots, r\}$ . These functions are shown in Table C1 for a single \$1 wager. Recall that the expected value of a function  $f_r$  of a random variable  $X$  defined by a discrete probability distribution can be calculated as

$$\mathbb{E}[f_r] = \sum_{i=0}^r f_r(x_i) \Pr(x_i; r).$$

This is essentially the mean (average) amount of return for playing  $r$  spots on a single \$1 wager. The conditional expected values in Table C2 are obtained by multiplying values in Table B2 with the corresponding entries in Table C1. As one can see in Table C2, the highest expected value is  $\mathbb{E}[X] = 0.65$  occurring when  $r = 7$ . It should be noted that all of the expected values are close to this value. One conclusion that can be inferred here is that on average, for every \$1 invested in a single base draw of KENO, a player will receive \$0.62 (averaging over all values of  $r$ :  $\bar{x} = 0.6200, s = 0.04443$ ), a \$0.38 loss!. Not necessarily the best investment strategy.

For any discrete probability distribution, one can compute the *Shannon entropy* of that distribution as

$$H(X) = - \sum_x p(x_i) \log_2 p(x_i) = \mathbb{E}[\log_2 p(X)].$$

Note that this quantity is determined by the probability distribution associated with  $X$ , not the values realized by  $X$ . Higher entropy indicates a greater level of uncertainty in predicting results of the game, which implies that players face more variability in outcomes. When comparing discrete distributions, lower values of  $H(X)$  are better.

The maximum Shannon entropy for a discrete distribution occurs when each outcome is equally likely. For example, suppose there are  $n$  possible outcomes, each occurring with probability  $1/n$ . Then

$$H_{\max}(X) = \log_2(n)$$

defines the maximum entropy for that that distribution. It will prove more useful to work with the entropy ratio  $H(X)/H_{\max}(X)$  which provides perhaps a better method of comparing entropies between distributions. As above, the lower values of this ratio suggests less uncertainty and more predictability.

### 3 Multiple Draws

Multiple draws in a KENO “play” occurs when one chooses  $D > 1$ , i.e.,  $D \in \{2, 3, 4, 5, 10, 20\}$ , where  $D$  is the number of consecutive draws to be played. The same numbers chosen, whether by player specification or by random quick picks, are used for all  $D$  draws.

We first assume that each draw is independent of past (and future) draws. This is not an unreasonable assumption because if the lottery game is fair, then this will be the case. In other words, numbers drawn in previous games will not affect the numbers to be drawn in the current game.

One way to think of discrete random variables are that they are variables that can take on numerical values, each with a specific probability. For a particular choice of  $r$ , we define the random variable  $X$  to represent the number of matches for a single draw, with the probabilities  $\Pr(x; r)$  calculated in B2 and the payoffs  $f_r(x)$  as tabulated in C1.

We first consider the case of two consecutive draws ( $D = 2$ ). Given that a choice has been made for  $r$ , let the random variable  $X$  represent the number of matches on the first draw with distribution function  $m_1(x)$ . Let random variable  $Y$  represent the number of matches on the second draw with distribution function  $m_2(y)$ .

We define the total winnings  $W$  for both draws as a new random variable given the number of spots being played is  $r$ :

$$W := f_r(X) + f_r(Y).$$

However, this says nothing about the probability distribution of the random variable  $W$ ,  $m_3(w)$ .

It turns out that the probability distribution of the sum of two independent discrete random variables is the convolution<sup>3</sup> of their individual distributions. Thus, the function  $m_3(w) = (m_1 * m_2)(w)$  is the distribution function of the random variable  $W = f_r(X) + f_r(Y)$ , the sum of two random variables.

We can easily extend this to more than two draws. Let  $W_D = \sum_{i=1}^D f_r(X_i) = \sum_{i=1}^D W(X_i)$  be the sum of  $D$  independent and identically distributed random variables with distribution function  $m$  defined on the integers. Then the distribution function of  $W_1$  is  $m$ , and

$$W_D = W_{D-1} + W_1,$$

with distribution function calculated as the convolution  $m_D = m_{D-1} * m$ .

It is important to realize here that for every unique combination of  $r$  and  $D$ , there is a unique random variable  $W_{r,D}$  with distribution function  $m_{r,D}(x)$ , which describes the sum of the winnings from

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<sup>3</sup>If  $X$  and  $Y$  are discrete random variables with probability mass functions  $m_X$  and  $m_Y$ , respectively, then the convolution of  $m_X$  and  $m_Y$  is defined as

$$(m_X * m_Y)(k) = \sum_i m_X(i) m_Y(k-i) = \sum_j m_X(k-j) m_Y(j).$$

all  $D$  consecutive draws. The payoffs for  $D$  consecutive draws, or a single card, are contained in  $W_{r,D}$ . These distributions are derived from the original distributions used for  $D = 1$ , i.e., the Hypergeometric Distribution.

For example, consider the cases where  $r = 1$ , and  $D \in \{1, 2, 3\}$ . The matches, payoffs, and probabilities are shown in Table D1. The cases for  $r = 3$ ,  $D \in \{1, 2, 3\}$  are shown in Table D2. These distributions are also displayed visually for  $r \leq 4$  in Figures D1 and D2. It is clear that as both  $r$  and  $D$  increase, the number of possible values for the sums  $W_{r,D}$  increases. This is illustrated more clearly in Table D3. To clarify, for the case with  $r = 4$  and  $D = 10$ , there are  $|W_{4,10}| = 230$  possible outcomes.

We are now ready to calculate probabilities for outcomes specific to questions of interest.  $W$  represents the sum of winnings from a single card. What is  $\Pr(W > 0)$ , i.e., what is the probability one wins money (no matter how small)? What is the net gain probability  $\Pr(W > D)$ , the probability that one gets more money back than what was wagered? Answering these questions is simply a matter of adding up individual probabilities from the  $m_{r,D}$  distributions. Results are summarized in Tables E1 and E2.

Another question of interest is a situation that happens when one wins something, but it isn't much (less than the amount that was wagered). We term this *bottom feeding*, and define it by the compound event  $(W > 0) \wedge (W \leq D)$ . These probabilities are summarized in Table E3. It should also be noted that  $\Pr(W > 0) = \Pr(W > 0 \wedge W \leq D) + \Pr(W > D)$ , i.e., win money probability can be decomposed into bottom feeding and net gain probabilities.

## 4 Analysis

We are now in a position to develop a strategy for playing KENO. Our primary goals are:

1. Win Money ( $W > 0$ ),
2. Have a Net Gain, i.e., win more than the amount wagered ( $W > D$ ),
3. Maximize the expected value  $E[W]$  of winnings,
4. Win large amounts for rare events (higher  $r$  is better),
5. Minimize uncertainty (Shannon Entropy ratio), and
6. Minimize bottom feeding ( $W > 0 \wedge W \leq D$ ).

Items 1,2,3,4 and 6 all involve both the probability distributions and payoff tables in their calculations. Item 5 only involves the probability distributions.

Within each choice of draw  $D$ , the numbers associated with each of the goals mentioned above were ranked from “best” to “worst”. Using the *Borda count method*<sup>4</sup> overall ranks were calculated and are tabulated in Table G1 and visually presented in Figure 2.

Note that these rank comparisons were made only within each  $D$ , and thus should not be compared across draws.

Consider  $D = 10$  draws. In Georgia KENO, it will take approximately thirty minutes for all 10 draws to complete. Within  $D = 10$ , the “best” choice for number of spots  $r$  to play (according to my ranking aggregation) is  $r = 8$ . Key probabilities and statistics for this choice of  $r$  and  $D$  are:

- Win Money:  $\Pr(W > 0) = 0.660$  (Table E1)
- Net Gain:  $\Pr(W > D) = 0.121$  (Table E2)
- Expected Value:  $E[W] = 6.47$  (Table C2, multiplied by  $D = 10$ )
- Uncertainty:  $H(X)/H_{\max}(X) = 0.227$  (Table F2)
- Bottom Feeding:  $\Pr(W > 0 \wedge W \leq D) = 0.540$ . (Table E3)

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<sup>4</sup>The Borda count method is named after the 18th-century French mathematician and naval engineer Jean-Charles de Borda, who re-devised the system in 1770. In a set of different rankings over the same population, it assigns maximum points to the “best” and 1 point to the “worst” within each set of rankings. To obtain a total ranking, these points are summed up, with the “best overall” determined by the maximum sum.

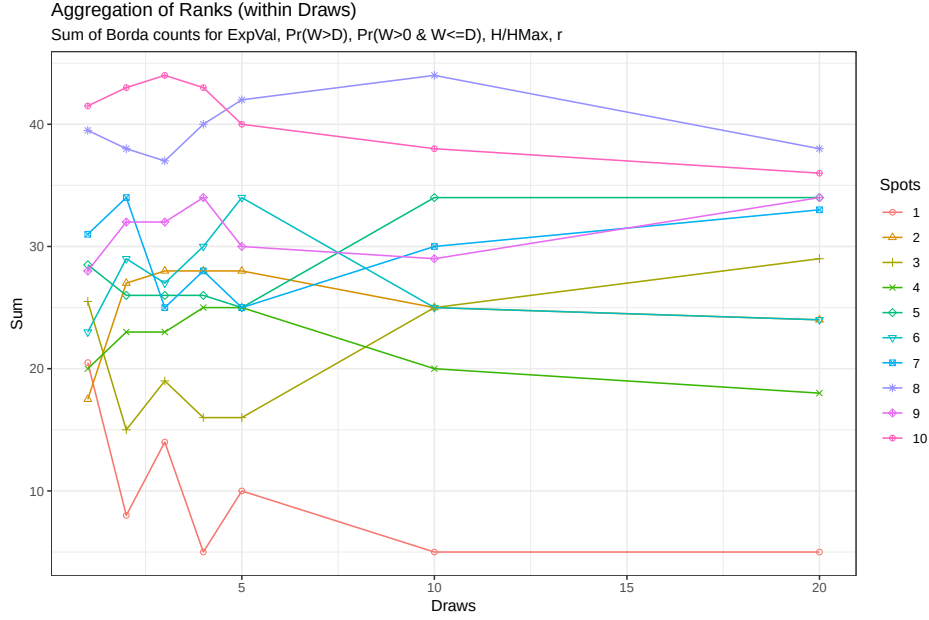


Figure 2: Aggregate Ranking Within Draws

To summarize, when playing a card with  $r = 8$  spots and  $D = 10$  draws, there is a 66% chance of winning money (and thus a 34% chance of not winning anything.) There is a 12% chance of achieving a net gain, i.e.,  $W > D$ . There is a 54% chance of bottom feeding, i.e., winning some money but not much. Note that the bottom feeding probability is third lowest of all choices of  $r$  given that there are 10 draws. Finally, the Shannon Entropy Ratio is the lowest of all  $r$  for  $r = 8$ , which implies that of all choices of  $r$  for  $D = 10$ ,  $r = 8$  minimizes uncertainty or variability in outcomes.

## 5 Final Thoughts

First and foremost, playing KENO is in the long run a losing proposition. The odds are just not that good. For example, a single base play with  $r = 7$  can expect to return \$0.65 on a \$1 wager (Table C2).

Second, playing consecutive multiple draws effectively serves as a “multiplier” on the probabilities of winning (not the payoffs of winning). For example, consider a single base wager of \$1 with  $r = 7$ . Referring to Tables E1 and E2, the probability of winning money is 23.7%. When playing 10 consecutive draws ( $D = 10$ ) with  $r = 7$  for a \$10 wager one has a 93.3% chance of winning money and a 9.6% chance of achieving a net gain.

Comparing  $r = 7$  and  $r = 8$  shows that  $r = 7$  has one of the best probabilities for getting something back, but is middle of the pack for probabilities of getting back a net gain.  $r = 8$  on the other hand has middle of the pack probability that one wins money, but is close to top of the pack for obtaining a net gain.

However, some substantial questions do remain. Are the draws truly independent of each other? Are the draws truly random? How accurate are these theoretical probabilities and expected values compared to actually playing KENO? Hopefully I will be able to answer some of these questions in the future.

Finally, while the probability calculations are exact, they do not allow one to predict what will occur on any particular KENO ticket. These probability calculations provide one with a picture of the probability landscape and given one’s wagers, the best possible opportunity to be successful. The only way these win probabilities can be realized is to play a large number of KENO tickets. Where this strategy pays off is if one is lucky enough to hit one of the big money, rare events. Sometimes one has to spend money to make money.

# A Georgia KENO

**GEORGIA LOTTERY**  
**KENO!**  
 with **BULLS-EYE** and **MULTIPLIER**

**1** How many numbers (spots) do you want to play?  
 1 2 3 4 5 6 7 8 9 10  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

**2** Pick your own numbers, OR choose Quik Pik.  
 1 2 3 4 5 6 7 8 9 10  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐  
 11 12 13 14 15 16 17 18 19 20  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐  
 21 22 23 24 25 26 27 28 29 30  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐  
 31 32 33 34 35 36 37 38 39 40  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐  
 41 42 43 44 45 46 47 48 49 50  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐  
 51 52 53 54 55 56 57 58 59 60  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐  
 61 62 63 64 65 66 67 68 69 70  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐  
 71 72 73 74 75 76 77 78 79 80  
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

**3** How much do you want to play per draw?  
 \$1 ☐ \$2 ☐ \$3 ☐ \$5 ☐ \$10 ☐

**4** How many consecutive draws do you want to play?  
 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 10 ☐ 20 ☐  
 Base Ticket Cost: Step 3 times Step 4

**MORE WAYS TO PLAY & WIN!**  

**Do you want to play MULTIPLIER?**  
 Play MULTIPLIER to increase KENO prizes by as much as 10 times! If you play MULTIPLIER, total ticket price increases by Base Ticket Cost from Step 4.

**Do you want to play BULLS-EYE?**  
 Play BULLS-EYE for bigger prizes and more chances to win! If you play BULLS-EYE, total ticket price increases by Base Ticket Cost from Step 4.

YES ☐ NO ☐

If both MULTIPLIER and BULLS-EYE options are played, total price is Base Ticket Cost times Seven. BULLS-EYE PRIZES CANNOT BE MULTIPLIED.

Figure A1: KENO Card (front)

| KENO! PRIZE PAYOUTS – PRIZES WON (PER \$1 PLAY) |            |                 |            |                 |            |                 |            |                 |            |                 |
|-------------------------------------------------|------------|-----------------|------------|-----------------|------------|-----------------|------------|-----------------|------------|-----------------|
| NUMBERS MATCHED                                 | 10 SPOT**  |                 | 9 SPOT**   |                 | 8 SPOT**   |                 | 7 SPOT     |                 | 6 SPOT     |                 |
|                                                 | Base Prize | Bulls-Eye Prize | Base Prize | Bulls-Eye Prize | Base Prize | Bulls-Eye Prize | Base Prize | Bulls-Eye Prize | Base Prize | Bulls-Eye Prize |
| 10                                              | \$100,000  | \$200,000       | \$30,000   | \$50,000        |            |                 |            |                 |            |                 |
| 9                                               | \$5,000    | \$20,000        | \$3,000    | \$10,000        | \$10,000   | \$40,000        |            |                 |            |                 |
| 8                                               | \$500      | \$1,500         | \$300      | \$900           | \$500      | \$900           | \$4,000    | \$6,000         |            |                 |
| 7                                               | \$50       | \$100           | \$150      | \$300           | \$500      | \$900           | \$125      | \$275           | \$1,200    | \$1,800         |
| 6                                               | \$10       | \$25            | \$25       | \$45            | \$75       | \$125           | \$15       | \$65            | \$50       | \$90            |
| 5                                               | \$2        | \$5             | \$5        | \$15            | \$10       | \$20            | \$3        | \$12            | \$7        | \$23            |
| 4                                               | \$1        | \$3             | \$1        | \$4             | \$2        | \$8             | \$1        | \$4             | \$1        | \$9             |
| 3                                               | \$1        | \$2             | \$1        | \$2             | \$1        | \$2             | \$1        | \$2             | \$1        | \$2             |
| 2                                               | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             |
| 1                                               | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             |
| 0                                               | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             |
| 0005                                            | 1 in 9.35  | 1 in 8.00       | 1 in 6.53  | 1 in 5.89       | 1 in 5.77  | 1 in 10.00      | 1 in 4.23  | 1 in 11.43      | 1 in 6.19  | 1 in 13.33      |
| NUMBERS MATCHED                                 | 5 SPOT     |                 | 4 SPOT     |                 | 3 SPOT     |                 | 2 SPOT     |                 | 1 SPOT     |                 |
|                                                 | Base Prize | Bulls-Eye Prize | Base Prize | Bulls-Eye Prize | Base Prize | Bulls-Eye Prize | Base Prize | Bulls-Eye Prize | Base Prize | Bulls-Eye Prize |
| 5                                               | \$400      | \$600           |            |                 |            |                 |            |                 |            |                 |
| 4                                               | \$17       | \$53            | \$70       | \$280           |            |                 |            |                 |            |                 |
| 3                                               | \$2        | \$13            | \$5        | \$20            | \$25       | \$100           |            |                 |            |                 |
| 2                                               | \$1        | \$5             | \$1        | \$11            | \$2        | \$18            | \$10       | \$55            | \$2        | \$48            |
| 1                                               | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             |
| 0                                               | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             | \$1        | \$1             |
| 0005                                            | 1 in 10.34 | 1 in 16.00      | 1 in 5.86  | 1 in 20.00      | 1 in 6.55  | 1 in 26.67      | 1 in 16.63 | 1 in 40.00      | 1 in 4.00  | 1 in 80.00      |

\* Subject to Georgia Lottery Rules, the prize amount won may be pari-mutuel. The total liability for 10 of 10 prize is limited to \$5,000,000.

\*\* Subject to Georgia Lottery Rules, prize amounts may be pari-mutuel. The total KENO! MULTIPLIER prize liability is limited to: 10 of 10 prize level - \$10 Millions; 9 of 9 prize level - \$3 Millions; 8 of 8 prize level - \$1 Millions.

**HOW TO PLAY KENO! MULTIPLIER**

KENO! MULTIPLIER gives you the chance to increase your KENO! winnings up to 10 times! Mark "YES" in the appropriate box. Your total ticket price increases by Base Ticket Cost from Step 4 if you add the MULTIPLIER option. A MULTIPLIER number will be drawn prior to each KENO! drawing. Nine, 0X, 5X, 10X, or 10X. If you played the MULTIPLIER option and your KENO! ticket is a winner, your prize will be multiplied by the selected MULTIPLIER for that draw. Overall odds of a MULTIPLIER number being 2X or higher are 1 in 1.33. The MULTIPLIER feature does not apply to BULLS-EYE prizes.

**HOW TO PLAY KENO! BULLS-EYE**

KENO! BULLS-EYE gives you bigger prizes and more chances to win! Mark "YES" in the appropriate box. Your total ticket price increases by Base Ticket Cost from Step 4 if you add the BULLS-EYE option. For every KENO! draw, one BULLS-EYE number will be randomly selected from the 20 winning numbers. If you played the BULLS-EYE option and match the BULLS-EYE number, you win the BULLS-EYE Prize in addition to the appropriate KENO! Base Prize for that spot game played. There are 3 ways to win: 1) match only the BULLS-EYE number drawn without any other KENO! number matches; 2) match your numbers to both the BULLS-EYE number and additional KENO! numbers drawn; or 3) match your KENO! numbers to those drawn without matching the BULLS-EYE number.

EXAMPLE: \$1 wager with a winning 4-spot game + BULLS-EYE match:

| Base Prize | Bulls-Eye Prize | Total Winnings |
|------------|-----------------|----------------|
| \$10       | \$100           | \$110          |

**HOW TO CLAIM YOUR PRIZE**

- Sign the back of the ticket. The ticket is a bearer instrument until signed.
- Prizes must be claimed within 180 days after the winning draw date. Date of claim is date of receipt by GLC.
- Present winning tickets of \$600 or less to any Georgia Lottery Retailer for validation and payment of prize.
- To claim prizes over \$600, present the ticket to any Georgia Lottery office, along with a valid government issued photo ID and social security card.
- Prizes can also be claimed by mailing signed ticket to: Georgia Lottery, P.O. Box 36966, Atlanta, GA 30363.
- For tickets or shares purchased using a registered FORTeCard, check your Players Club Account for prize payment information.
- The Lottery Headquarters address is 250 Wilshire Street, Suite 3000, Atlanta, GA 30303; for additional assistance please call 1-800-GA-LUCKY.

**KENO! RULES**

- KENO! tickets may be canceled only on the day of purchase, prior to the first draw on the ticket, at the retailer location where they were purchased. Exchange tickets cannot be canceled.
- This play is neither valid receipt nor proof of purchase. The printed lottery ticket is the only valid receipt and proof of purchase for claiming a prize.
- The holder of a winning KENO! ticket wins only one prize per drawing in connection with winning numbers drawn, but shall be entitled to the highest prize won by those numbers.
- All official KENO! tickets purchased from the Georgia Lottery Corporation are governed by applicable state law, and the Rules and Regulations of the Georgia Lottery. By submitting a KENO! play slip for processing, the player agrees to abide by the Georgia Lottery Rules and Regulations.
- State law prohibits the sale of any lottery ticket to a person under 18 years of age.
- The Georgia Lottery Corporation is not responsible for lost, stolen, or mutilated lottery tickets.

For assistance with problem gaming, please contact Helpline at 1-800-522-4780.

PLAY RESPONSIBLY – It's all about fun!  
Visit our website at [www.galottery.com](http://www.galottery.com)

GA-9007-02 FG 06/14 1/2

Figure A2: KENO Card (back)

## B KENO Base Probability

| $r$ | $E[X] = \mu_r$ | $\text{Var}[X] = \sigma_r^2$ | $\sqrt{\text{Var}[X]} = \sigma_r$ | $H_r(X)$ |
|-----|----------------|------------------------------|-----------------------------------|----------|
| 1   | 0.25           | 0.625                        | 0.791                             | 0.811    |
| 2   | 0.5            | 1.234                        | 1.111                             | 1.24     |
| 3   | 0.75           | 1.828                        | 1.352                             | 1.53     |
| 4   | 1              | 2.405                        | 1.551                             | 1.74     |
| 5   | 1.25           | 2.967                        | 1.722                             | 1.91     |
| 6   | 1.5            | 3.513                        | 1.874                             | 2.04     |
| 7   | 1.75           | 4.043                        | 2.011                             | 2.15     |
| 8   | 2              | 4.557                        | 2.135                             | 2.24     |
| 9   | 2.25           | 5.055                        | 2.248                             | 2.33     |
| 10  | 2.5            | 5.538                        | 2.353                             | 2.40     |

Table B1: Hypergeometric Distribution [Hyp(80, $r$ ,20)] Properties

| Match \ Spots | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       |
|---------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0             | 7.50e-01 | 5.60e-01 | 4.17e-01 | 3.08e-01 | 2.27e-01 | 1.67e-01 | 1.22e-01 | 8.83e-02 | 6.37e-02 | 4.58e-02 |
| 1             | 2.50e-01 | 3.80e-01 | 4.31e-01 | 4.33e-01 | 4.06e-01 | 3.63e-01 | 3.15e-01 | 2.66e-01 | 2.21e-01 | 1.80e-01 |
| 2             |          | 6.01e-02 | 1.39e-01 | 2.13e-01 | 2.70e-01 | 3.08e-01 | 3.27e-01 | 3.28e-01 | 3.16e-01 | 2.95e-01 |
| 3             |          |          | 1.39e-02 | 4.32e-02 | 8.39e-02 | 1.30e-01 | 1.75e-01 | 2.15e-01 | 2.46e-01 | 2.67e-01 |
| 4             |          |          |          | 3.06e-03 | 1.21e-02 | 2.85e-02 | 5.22e-02 | 8.15e-02 | 1.14e-01 | 1.47e-01 |
| 5             |          |          |          |          | 6.45e-04 | 3.10e-03 | 8.64e-03 | 1.83e-02 | 3.26e-02 | 5.14e-02 |
| 6             |          |          |          |          |          | 1.29e-04 | 7.32e-04 | 2.37e-03 | 5.72e-03 | 1.15e-02 |
| 7             |          |          |          |          |          |          | 2.44e-05 | 1.60e-04 | 5.92e-04 | 1.61e-03 |
| 8             |          |          |          |          |          |          |          | 4.35e-06 | 3.26e-05 | 1.35e-04 |
| 9             |          |          |          |          |          |          |          |          | 7.24e-07 | 6.12e-06 |
| 10            |          |          |          |          |          |          |          |          |          | 1.12e-07 |

Table B2: KENO Base Probability  $\Pr(x; 80, r, 20)$ ,  $r = 1, 2, \dots, 10$

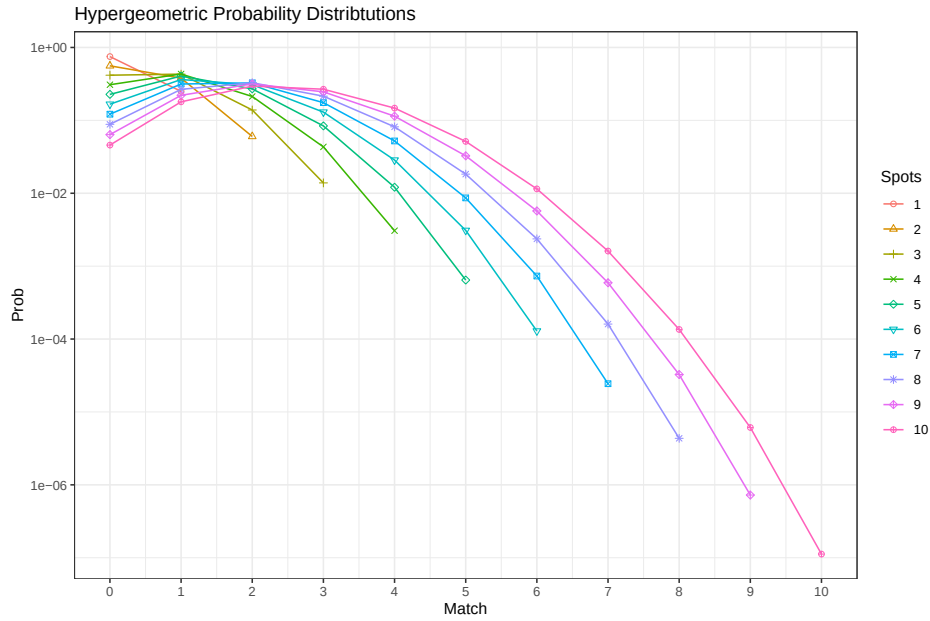


Figure B1: KENO Probability Distributions (defined only at integer Spots Hit  $x$ )

## C KENO Base Payoff and Expected Values

| Payoff Function<br>Match \ Spots | $f_1$<br>1 | $f_2$<br>2 | $f_3$<br>3 | $f_4$<br>4 | $f_5$<br>5 | $f_6$<br>6 | $f_7$<br>7 | $f_8$<br>8 | $f_9$<br>9 | $f_{10}$<br>10 |
|----------------------------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|----------------|
| 0                                | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 5              |
| 1                                | 2          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0              |
| 2                                |            | 10         | 2          | 1          | 0          | 0          | 0          | 0          | 0          | 0              |
| 3                                |            |            | 25         | 5          | 2          | 1          | 1          | 0          | 0          | 0              |
| 4                                |            |            |            | 70         | 17         | 7          | 3          | 2          | 1          | 0              |
| 5                                |            |            |            |            | 400        | 50         | 15         | 10         | 5          | 2              |
| 6                                |            |            |            |            |            | 1200       | 125        | 75         | 25         | 10             |
| 7                                |            |            |            |            |            |            | 4000       | 500        | 150        | 50             |
| 8                                |            |            |            |            |            |            |            | 10000      | 3000       | 500            |
| 9                                |            |            |            |            |            |            |            |            | 30000      | 5000           |
| 10                               |            |            |            |            |            |            |            |            |            | 100000         |

Table C1: KENO Base Payoff Functions  $f_r(x)$ ,  $r = 1, 2, \dots, 10$

| Match \ Spots  | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       |
|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0              |          |          |          |          |          |          |          |          |          | 2.29e-01 |
| 1              | 5.00e-01 |          |          |          |          |          |          |          |          |          |
| 2              |          | 6.01e-01 | 2.78e-01 | 2.13e-01 |          |          |          |          |          |          |
| 3              |          |          | 3.47e-01 | 2.16e-01 | 1.68e-01 | 1.30e-01 | 1.75e-01 |          |          |          |
| 4              |          |          |          | 2.14e-01 | 2.06e-01 | 2.00e-01 | 1.57e-01 | 1.63e-01 | 1.14e-01 |          |
| 5              |          |          |          |          | 2.58e-01 | 1.55e-01 | 1.30e-01 | 1.83e-01 | 1.63e-01 | 1.03e-01 |
| 6              |          |          |          |          |          | 1.55e-01 | 9.15e-02 | 1.78e-01 | 1.43e-01 | 1.15e-01 |
| 7              |          |          |          |          |          |          | 9.76e-02 | 8.02e-02 | 8.88e-02 | 8.06e-02 |
| 8              |          |          |          |          |          |          |          | 4.35e-02 | 9.78e-02 | 6.77e-02 |
| 9              |          |          |          |          |          |          |          |          | 2.17e-02 | 3.06e-02 |
| 10             |          |          |          |          |          |          |          |          |          | 1.12e-02 |
| Expected Value | 5.00e-01 | 6.01e-01 | 6.24e-01 | 6.43e-01 | 6.31e-01 | 6.39e-01 | 6.50e-01 | 6.47e-01 | 6.28e-01 | 6.37e-01 |

Table C2: Conditional Expected Values

## D Derived Random Variable $W$ Distributions

| $W_{1,1}$ | $m_{1,1}$ | $W_{1,2}$ | $m_{1,2}$ | $W_{1,3}$ | $m_{1,3}$ |
|-----------|-----------|-----------|-----------|-----------|-----------|
| 0         | 0.75      | 0         | 0.5625    | 0         | 0.421875  |
| 2         | 0.25      | 2         | 0.3750    | 2         | 0.421875  |
|           |           | 4         | 0.0625    | 4         | 0.140625  |
|           |           |           |           | 6         | 0.015625  |

Table D1: Derived Distributions for  $r = 1$ ,  $D \in \{1, 2, 3\}$

| $W_{3,1}$ | $m_{3,1}$ | $W_{3,2}$ | $m_{3,2}$ | $W_{3,3}$ | $m_{3,3}$ |
|-----------|-----------|-----------|-----------|-----------|-----------|
| 0         | 0.417     | 0         | 0.718     | 0         | 0.608     |
| 0         | 0.431     | 2         | 0.235     | 2         | 0.299     |
| 2         | 0.139     | 4         | 0.0193    | 4         | 0.0489    |
| 25        | 0.0139    | 25        | 0.0235    | 6         | 2.67e-03  |
|           |           | 27        | 0.00385   | 25        | 2.99e-02  |
|           |           | 50        | 0.000193  | 27        | 9.79e-03  |
|           |           |           |           | 29        | 8.01e-04  |
|           |           |           |           | 50        | 4.89e-04  |
|           |           |           |           | 52        | 8.01e-05  |
|           |           |           |           | 75        | 2.67e-06  |

Table D2: Derived Distributions for  $r = 3$ ,  $D \in \{1, 2, 3\}$

| $r$ | $D = 1$ | $D = 2$ | $D = 3$ | $D = 4$ | $D = 5$ | $D = 10$ | $D = 20$ |
|-----|---------|---------|---------|---------|---------|----------|----------|
| 1   | 2       | 3       | 4       | 5       | 6       | 11       | 21       |
| 2   | 3       | 3       | 4       | 5       | 6       | 11       | 21       |
| 3   | 4       | 6       | 10      | 15      | 21      | 66       | 231      |
| 4   | 5       | 10      | 20      | 35      | 55      | 230      | 875      |
| 5   | 6       | 10      | 20      | 35      | 56      | 286      | 1751     |
| 6   | 7       | 15      | 35      | 70      | 126     | 941      | 6181     |
| 7   | 8       | 21      | 55      | 120     | 230     | 1973     | 14339    |
| 8   | 9       | 21      | 56      | 126     | 251     | 2751     | 33354    |
| 9   | 10      | 28      | 84      | 210     | 460     | 6874     | 93874    |
| 10  | 11      | 35      | 112     | 294     | 670     | 11668    | 210955   |

Table D3: Length of Derived Distributions  $|W_{r,D}|$

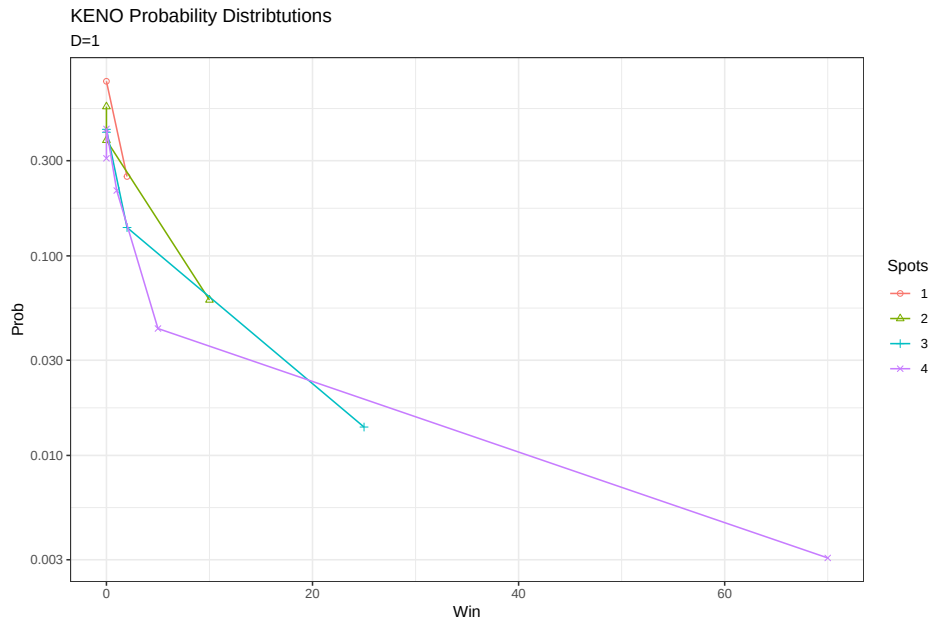


Figure D1: Derived Probability Mass Functions,  $r = 1, \dots, 4$ ,  $D = 1$

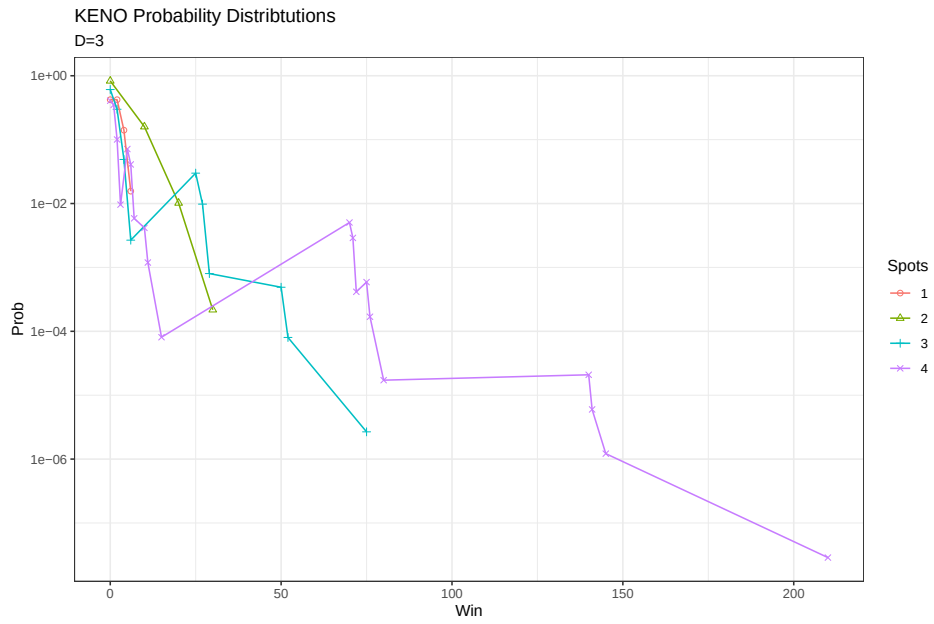


Figure D2: Derived Probability Mass Functions,  $r = 1, \dots, 4$ ,  $D = 3$

## E Probabilities

| $r$ | $D = 1$ | $D = 2$ | $D = 3$ | $D = 4$ | $D = 5$ | $D = 10$ | $D = 20$ |
|-----|---------|---------|---------|---------|---------|----------|----------|
| 1   | 0.2500  | 0.437   | 0.578   | 0.684   | 0.763   | 0.944    | 0.997    |
| 2   | 0.0601  | 0.117   | 0.170   | 0.220   | 0.267   | 0.462    | 0.711    |
| 3   | 0.1526  | 0.282   | 0.392   | 0.484   | 0.563   | 0.809    | 0.964    |
| 4   | 0.2589  | 0.451   | 0.593   | 0.698   | 0.777   | 0.950    | 0.998    |
| 5   | 0.0967  | 0.184   | 0.263   | 0.334   | 0.399   | 0.638    | 0.869    |
| 6   | 0.1616  | 0.297   | 0.411   | 0.506   | 0.586   | 0.828    | 0.971    |
| 7   | 0.2366  | 0.417   | 0.555   | 0.660   | 0.741   | 0.933    | 0.995    |
| 8   | 0.1023  | 0.194   | 0.277   | 0.351   | 0.417   | 0.660    | 0.885    |
| 9   | 0.1531  | 0.283   | 0.392   | 0.485   | 0.564   | 0.810    | 0.964    |
| 10  | 0.1105  | 0.209   | 0.296   | 0.374   | 0.443   | 0.690    | 0.904    |

Table E1: Win Money:  $\Pr(W > 0)$

| $r$ | $D = 1$ | $D = 2$ | $D = 3$ | $D = 4$ | $D = 5$ | $D = 10$ | $D = 20$ |
|-----|---------|---------|---------|---------|---------|----------|----------|
| 1   | 0.2500  | 0.0625  | 0.1563  | 0.0508  | 0.1035  | 0.0197   | 0.00394  |
| 2   | 0.0601  | 0.1166  | 0.1698  | 0.2197  | 0.2666  | 0.1180   | 0.11552  |
| 3   | 0.1526  | 0.0468  | 0.0927  | 0.0638  | 0.0883  | 0.1313   | 0.24382  |
| 4   | 0.0463  | 0.0905  | 0.1326  | 0.1728  | 0.1459  | 0.0893   | 0.07520  |
| 5   | 0.0967  | 0.0324  | 0.0574  | 0.0522  | 0.0671  | 0.1204   | 0.12924  |
| 6   | 0.0318  | 0.0625  | 0.0923  | 0.1211  | 0.1490  | 0.0674   | 0.08053  |
| 7   | 0.0616  | 0.1194  | 0.0824  | 0.0677  | 0.0744  | 0.0960   | 0.10976  |
| 8   | 0.1023  | 0.0479  | 0.0796  | 0.0828  | 0.1045  | 0.1206   | 0.09004  |
| 9   | 0.0389  | 0.0764  | 0.1123  | 0.1469  | 0.0963  | 0.0872   | 0.12260  |
| 10  | 0.1105  | 0.1172  | 0.1740  | 0.2165  | 0.1200  | 0.1167   | 0.09399  |

Table E2: Net Gain:  $\Pr(W > D)$

| $r$ | $D = 1$ | $D = 2$ | $D = 3$ | $D = 4$ | $D = 5$ | $D = 10$ | $D = 20$ |
|-----|---------|---------|---------|---------|---------|----------|----------|
| 1   | 0.000   | 0.3750  | 0.422   | 0.633   | 0.659   | 0.924    | 0.993    |
| 2   | 0.000   | 0.0000  | 0.000   | 0.000   | 0.000   | 0.344    | 0.595    |
| 3   | 0.000   | 0.2352  | 0.299   | 0.421   | 0.475   | 0.678    | 0.720    |
| 4   | 0.213   | 0.3604  | 0.460   | 0.526   | 0.631   | 0.861    | 0.922    |
| 5   | 0.000   | 0.1516  | 0.205   | 0.282   | 0.331   | 0.518    | 0.740    |
| 6   | 0.130   | 0.2345  | 0.318   | 0.385   | 0.437   | 0.761    | 0.890    |
| 7   | 0.175   | 0.2978  | 0.473   | 0.593   | 0.666   | 0.837    | 0.886    |
| 8   | 0.000   | 0.1463  | 0.197   | 0.268   | 0.313   | 0.540    | 0.795    |
| 9   | 0.114   | 0.2063  | 0.280   | 0.339   | 0.468   | 0.723    | 0.841    |
| 10  | 0.000   | 0.0915  | 0.122   | 0.157   | 0.323   | 0.573    | 0.810    |

Table E3: Bottom Feeding:  $\Pr(W > 0 \wedge W \leq D)$

## F Shannon Entropy $H(X)$

| $r$ | $D = 1$ | $D = 2$ | $D = 3$ | $D = 4$ | $D = 5$ | $D = 10$ | $D = 20$ |
|-----|---------|---------|---------|---------|---------|----------|----------|
| 1   | 0.811   | 1.248   | 1.542   | 1.762   | 1.936   | 2.47     | 2.99     |
| 2   | 1.243   | 0.543   | 0.715   | 0.861   | 0.989   | 1.46     | 2.01     |
| 3   | 1.531   | 1.104   | 1.424   | 1.686   | 1.908   | 2.69     | 3.59     |
| 4   | 1.743   | 1.630   | 2.081   | 2.443   | 2.741   | 3.67     | 4.50     |
| 5   | 1.908   | 0.856   | 1.128   | 1.358   | 1.560   | 2.31     | 3.24     |
| 6   | 2.041   | 1.260   | 1.640   | 1.956   | 2.228   | 3.21     | 4.34     |
| 7   | 2.151   | 1.670   | 2.128   | 2.476   | 2.753   | 3.62     | 4.48     |
| 8   | 2.245   | 0.939   | 1.242   | 1.501   | 1.729   | 2.59     | 3.60     |
| 9   | 2.325   | 1.273   | 1.669   | 2.002   | 2.291   | 3.34     | 4.45     |
| 10  | 2.395   | 1.116   | 1.482   | 1.795   | 2.071   | 3.11     | 4.34     |

Table F1: Shannon Entropy  $H(X)$

| $r$ | $D = 1$ | $D = 2$ | $D = 3$ | $D = 4$ | $D = 5$ | $D = 10$ | $D = 20$ |
|-----|---------|---------|---------|---------|---------|----------|----------|
| 1   | 0.811   | 0.787   | 0.771   | 0.759   | 0.749   | 0.715    | 0.681    |
| 2   | 0.784   | 0.343   | 0.358   | 0.371   | 0.383   | 0.422    | 0.458    |
| 3   | 0.765   | 0.427   | 0.429   | 0.431   | 0.434   | 0.445    | 0.457    |
| 4   | 0.751   | 0.491   | 0.482   | 0.476   | 0.474   | 0.468    | 0.461    |
| 5   | 0.738   | 0.258   | 0.261   | 0.265   | 0.269   | 0.284    | 0.301    |
| 6   | 0.727   | 0.322   | 0.320   | 0.319   | 0.319   | 0.325    | 0.344    |
| 7   | 0.717   | 0.380   | 0.368   | 0.358   | 0.351   | 0.331    | 0.324    |
| 8   | 0.708   | 0.214   | 0.214   | 0.215   | 0.217   | 0.227    | 0.239    |
| 9   | 0.700   | 0.265   | 0.261   | 0.259   | 0.259   | 0.262    | 0.270    |
| 10  | 0.692   | 0.218   | 0.218   | 0.219   | 0.221   | 0.230    | 0.245    |

Table F2: Shannon Entropy Ratio  $H(X)/H_{\max}(X)$

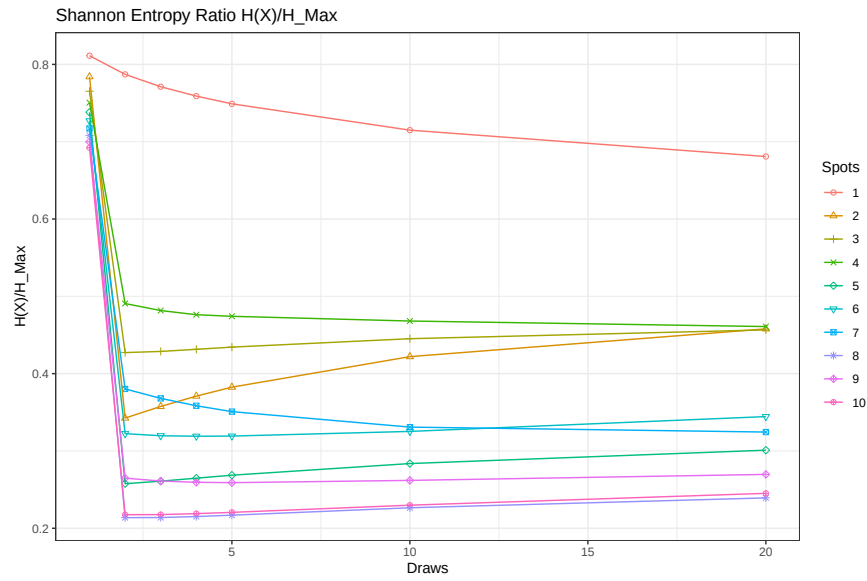


Figure F1: Shannon Entropy Ratio  $H(X)/H_{\max}(X)$

## G Aggregate Ranking

| $r$        | $D = 1$ | $D = 2$ | $D = 3$ | $D = 4$ | $D = 5$ | $D = 10$ | $D = 20$ |
|------------|---------|---------|---------|---------|---------|----------|----------|
| 1          | 20.5    | 8       | 14      | 5       | 10      | 5        | 5        |
| 2          | 17.5    | 27      | 28      | 28      | 28      | 25       | 24       |
| 3          | 25.5    | 15      | 19      | 16      | 16      | 25       | 29       |
| 4          | 20.0    | 23      | 23      | 25      | 25      | 20       | 18       |
| 5          | 28.5    | 26      | 26      | 26      | 25      | 34       | 34       |
| 6          | 23.0    | 29      | 27      | 30      | 34      | 25       | 24       |
| 7          | 31.0    | 34      | 25      | 28      | 25      | 30       | 33       |
| 8          | 39.5    | 38      | 37      | 40      | 42      | 44       | 38       |
| 9          | 28.0    | 32      | 32      | 34      | 30      | 29       | 34       |
| 10         | 41.5    | 43      | 44      | 43      | 40      | 38       | 36       |
| “Best” $r$ | 10      | 10      | 10      | 10      | 8       | 8        | 8        |

Table G1: Aggregate Rank Totals